

# Chromoelectric response functions for quark-gluon plasma

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We determine the chromoelectric response of quark-gluon plasma (QGP) systematically within the framework of classical transport equations. The transport equations are set up in the phase space which includes the  $SU(3)$  group space corresponding to color (which is a dynamical degree of freedom), in addition to the position - momentum variables. The distribution functions are defined by projecting the density operators for the quarks and the gluons to their respective coherent states (defined over the extended phase space). The full import of the Yang-Mills (YM) dynamics is shown to manifest through the emergence of an intrinsic nonlinear, nonlocal response, whose behavior we determine in the long wavelength limit. It also manifests as a tensor response which is a characteristic of gluons. The response functions are shown to have a natural interpretation in terms of the renormalizations of the Abelian and the non-Abelian coupling constants. A detailed analysis of the screening of heavy quark potential and of the exact role played by the Debye mass screening in the case of the Cornell potential, is performed. We also discuss the non-Abelian contribution to Landau damping in QGP.

## I. INTRODUCTION

The purpose of this paper is to study systematically the color response functions of quark gluon plasma, with a proper incorporation of the non-Abelian dynamics. Considering the color electric case, we show the emergence of additional response functions which do not have counterparts in the well studied electrodynamic plasmas. More precisely, we show the emergence of a non-Abelian component of the chromoelectric permittivity in the matter and gluonic sectors. The gluonic sector is shown to exhibit yet another response, corresponding to its color-tensor excitations. In this work, we merely illustrate these responses in the simple case of ideal plasma. We pay particular attention to the role of screening length in modifying the dynamics of heavy quarkonium systems. Applications to more realistic cases will be taken up in a separate work. The analysis is performed within the framework of a classical transport equation – the analog of Vlasov equation – as is appropriate to non-Abelian dynamics. We start with the motivation below.

Searches for QGP in Ultra Relativistic Heavy Ion Collisions (URHIC) have been on experimentally [1, 2, 3, 4] for more than a decade. We have today a wealth of information coming from observations involving (i) particle multiplicities, (ii) minijets, (iii) jet quenching, (iv) strangeness enhancement, (v) collective flows, (vi) heavy quark dynamics ( $J/\Psi$  suppression)[5], (vii) Hanbury Brown-Twiss interferometric measurements[6] etc. It is strongly suspected that QGP is already produced in experiments at RHIC. Further, it has been inferred from flow measurements that the deconfined phase is, in all likelihood, a liquid state with a very low viscosity –

smaller than even that of liquid helium [7, 8]. The upcoming experiments at Large Hadron Collider (LHC) may be expected to reveal many more (unexpected) results.

A parallel development that has taken place theoretically is the realization that QGP exhibits a strong collective behavior which cannot be understood perturbatively [8, 9]. First of all, improved lattice computations [9, 10] predict that the deconfined phase close to the transition temperature  $T_c$  is highly non-perturbative, with the equation of state being far away from that of the ideal gas of quarks and gluons. The ideal behavior is not expected to be realized even at  $T = 2T_c$ . Analytic studies [11] based on improved perturbative approaches (with computations of  $O(g^6 \log(g))$ ) also arrive at the same conclusion and indicate that the ideal behavior will be seen only at much higher temperatures. Experimentally, the flow measurements at RHIC do indicate a low viscosity liquid like behavior of the QGP [12]. Finally, it is known by now [13, 14, 15] that a classical behavior emerges naturally when one considers hard thermal loop (HTL) contributions. A local formulation of HTL effective action has been obtained by Blaizot and Iancu who have succeeded in rewriting the HTL effective theory as a kinetic theory with a Vlasov term [16, 17, 18].

It may be, therefore, a fruitful endeavor to take the classical frame work seriously and explore the extent to which it can capture the expected features of QGP. It is well recognized that its success would depend crucially on the availability of reasonable mechanisms for (i) the production of soft partons (rate term in the phase space) and (ii) the expression for the equation of state and its equilibrium distribution function. Indeed, significant progress has been made in both the directions by employing a variety of techniques [19, 20, 21, 22], although they are all not always mutually consistent. Be it as may, we wish to point out in this paper that the evaluation of the response functions of the QGP, so crucial for the signals, is further dependent on the form of

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the distribution functions in the color space, which is inherited from the parent density operators; it may not be assigned at will.

The central theme of the paper is to couple the coherent state representation in the color space with the YM dynamics in the transport equations. In doing so, we derive, in a consistent manner, the response functions which show manifestly the Abelian and the non-Abelian contributions. The latter is nonlinear and nonlocal in space time. Hence it is also non-Markovian. We also show the intrinsic distinction between a pure QCD plasma which is purely gluonic and QGP which has both matter and gluonic components. Since we do not wish to make any realistic calculation here, many simplifications will be made in the transport equations: (i) the equilibrium distribution functions will be taken to be ideal, (ii) we study the response when the system is close to equilibrium, so that the source term may also be dropped and (iii) finally, we take the plasma to be spatially isotropic. The plasma is taken to be collisionless. These simplifications will be improved upon in a separate paper, where more realistic collision terms such as the one derived by Bodeker [23] will be used. As remarked above, the primary purpose of the paper is to elucidate the structure of the response functions and to illustrate it in simple situations.

The chromoelectric response functions are particularly important, ever since Matsui and Satz [24] predicted  $J/\Psi$  suppression as a signature for QGP in heavy ion collisions. Lattice calculations [9] do support the prediction; early estimates of the suppression in heavy ion collisions have involved the assumption of a hydrodynamic expansion of an ideal plasma. The suppression is of course determined by the Debye mass which evolves with the temperature. We study the precise role of the Debye mass in the specific case of the Cornell potential. In any case, it appears that the Debye mass is an incomplete manifestation of the non-Abelian dynamics, in as much as that it *does not* require the non-Abelian interactions for its emergence. It is not unreasonable to look for exclusively non-Abelian responses which would be nonlinear, and reflect additional properties of chromodynamics in a medium. We do show that such a non-Abelian permittivity exists. The new permittivity may also lead to other signatures which can perhaps be tested experimentally.

### A. A brief review

We briefly review the studies on the QGP response functions so far to the extent that they are relevant to our work. There have been many attempts to calculate the response function for QGP. An approach based on perturbative QCD (pQCD) at finite temperature was adopted by Weldon [25] who studied plasma screening and plasma oscillations. Based on an Abelian analysis, Mustafa, Thoma and Chakraborty [26] have argued for bound quark structures, which have also been predicted by Liao and Shuryak [27]. The conclusion is essentially

based on their analysis for moving partons. Petreczky [28] has, on the other hand, considered a  $SU(2)$  plasma, and extracted the chromoelectric screening by analyzing the long distance behavior of the static part of the longitudinal propagator in the lowest order. His results are off by about 25% from lattice simulations, and are not easily adapted to the scenario prevalent in URHIC. Finally, in a work somewhat close to ours, Chen *et al* [29] have employed the Vlasov equation to study the color response. However, the gauge invariance of their results is not apparent, as also its applicability to URHIC. None of the above approaches employs the coherent state projections which establish a natural connection between the underlying quantum dynamics with the classical approximations. They do not also study exhaustively *all possible* response functions involving the color degree of freedom.

## II. DETERMINATION OF THE RESPONSE FUNCTIONS

As mentioned above, the approach will be based on the transport equation for the phase space distribution functions for the quarks and the gluons. The distribution functions for an  $N$  particle system will have the generic form  $f(\vec{q}_1, \dots, \vec{q}_N; \vec{p}_1, \dots, \vec{p}_N; Q_1^a, \dots, Q_N^a)$  in terms of the coordinates  $\vec{q}$ , the momenta  $\vec{p}$ , and the color charges  $Q^a$  which are defined in the space corresponding to the gauge group. Note that the distribution function is invariant under gauge transformations (for a formal demonstration, see *e.g.*, [30]). Let us first consider the generic form of the transport equation

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial p_i} + \dot{Q}^a \frac{\partial f}{\partial Q^a} = \Sigma + C, \quad (1)$$

where the source term is denoted by  $\Sigma$  and  $C$  is the collision term. The source term is indeed important in studying the production and evolution of QGP [31, 32, 33]. But as mentioned above, we study the response when system is close to its equilibrium configuration, whence the source term may be dropped. We further work in the Vlasov limit so that the collision term will also be dropped. The responses are, of course eventually evaluated in the limit that the collision time  $\tau_c \rightarrow \infty$ , via the standard Landau prescription. The reason for working in this limit is that the inclusion of any realistic collision term, as, for instance, the one derived by Bodeker [23] will complicate the discussions. On the other hand, a simple minded relaxation term would violate current conservation [34]. This drawback will be remedied in a separate paper.

The third term in Eq.1 represents the Vlasov term corresponding to the action of the mean field on the plasma. The next term, involving the derivative of the color charge, is unique to the non-Abelian plasma, and gives the dynamical variation of the color charge (in the compact part of the phase space). The dynamics

modulates and modifies the contribution of the standard Vlasov term; further, it also gives rise to the non-Abelian component of the response function. Our analysis makes crucial use of the most general form of the quark and the gluonic distribution functions in the color variables, and we take up the determination of their form in the next section.

## A. The distribution functions

### 1. The coherent bases

We elucidate the procedure for obtaining the generic forms of phase space distribution functions from their underlying quantum states. The best way of extracting them is to project the parent quantum state to a coherent basis. Indeed, with the usual position-momentum variables, the coherent states are the closest to the classical states since they possess minimum uncertainty in position and momentum. The projection defines, in a natural manner, the area over which the phase space is coarse grained: it is simply given by  $2\pi\hbar$  for each degree of freedom. Further, the coherent state projection is equivalent to the Wigner distribution function used *e.g.* by Elze and Heinz [35, 36]. However, it has the advantage that unlike the Wigner distribution function which can take negative values, the phase space density obtained from the coherent state is always non-negative.

The above method needs a refinement when it comes to the color degree of freedom. Recall that the classical phase spaces associated with finite dimensional Hilbert spaces are compact (as for example, in the case of spin). There is no neat separation of canonically conjugate variables, and there is no associated commutator (of the kind  $[x, p] = i$ ) that leads to minimum uncertainty states. Instead, the commutators are given by the Lie Algebra of the associated compact group. Care needs to be taken in defining the phase space distribution in this case. The most convenient method is to follow Perelomov [37]: take a representative state, and act the group  $\exp\{iT_a\theta_a\}$  on a reference state which is typically taken to be the state with the highest weight. The resulting orbit,  $\mathcal{O}(|\theta_a\rangle)$  forms a faithful copy of the associated phase space. The quantity  $\langle\theta_1, \dots, \theta_n|\hat{\rho}|\theta_1, \dots, \theta_n\rangle$  is interpreted consistently as the classical probability density in the color part of the phase space. These distributions so obtained are always smeared, for any finite dimensional representation of the compact group.

### 2. Distribution functions for quarks and gluons

We employ the method outlined above for the specific system of interest. Let  $\hat{\rho}$  be the density operator for a parton. Denote by  $|\Psi\rangle_c = |\vec{r}, \vec{p}\rangle \otimes |Q^a\rangle$  the coherent basis for the parton, where the first term is the usual minimum uncertainty state. As mentioned, the latter

term is obtained by the action of the gauge group on a standard state,  $|\psi\rangle_s$ , taken to be the one with the highest weight [37]. Thus the coherent states in the color sector have the form  $|Q^a\rangle = \exp\{iQ^a T^a\}|\psi_s\rangle$ , where the variables  $Q^a$  provide a coordinate description of the group space. For example, the parameters for  $SU(2)$  may be chosen to be the Euler angles. A generalized Euler angle description for  $SU(3)$ , which is of relevance to us here has been provided by Byrd [38]. We will be using it subsequently.

The form of the generators  $T^a$  are to be chosen depending on the parton, *i.e.*, the representation to which it belongs. The quarks transform according to the fundamental representation of the gauge group and the generators will also be chosen accordingly. Gluons, on the other hand, belong to the adjoint representation. As a direct consequence, the dimension of the phase space will be six for the quarks, and eight for the gluons if we consider  $SU(3)$  gauge group. Incidentally, note that the phase space for the quark is isomorphic to its Hilbert space. The gluonic phase space constitutes, in contrast, a coset space, being an orbit of the gauge group in the Hilbert space which is of dimension fifteen. The property of the coherent state depends naturally on the orbit to which it belongs. Suffice to say, the color coherent states are natural generalizations of the more well known spin coherent states. We refer the reader to literature [37] for more details.

The structure of the copy of the phase space, obtained thus as a coset space has strong implications on the nature of the color distributions, bearing a deep connection with the classical-quantum correspondence. For any finite dimensional representation of the group, only a finite number of the moments of the variables can contribute. To illustrate with a simple example, the state of a spin half particle is characterized entirely by the vector polarization  $\text{Tr}\{\rho\vec{S}\}$ . A spin one state requires, in addition, a specification of the second order tensor polarization  $\text{Tr}\{\rho S_{ij}\}$ , where  $S_{ij} = \frac{S_i S_j + S_j S_i}{2} - \frac{1}{3} S^2 \delta_{ij}$ .

We perform a similar analysis for quarks and gluons. Since the quarks belong to the fundamental representation of the gauge group, the quark distributions can at the most be linear in the color charge  $Q$ . All the higher order terms vanish identically. Thus, the single quark distribution function has the most general form  $f = f_0 + f^a \hat{Q}^a$  where  $f_0, f^a$  are functions of the usual phase space variables  $\{x_i, p_i\}$ . In contrast, the gluons belong to the adjoint representation; they admit a more general expansion, with an additional bilinear term in the color charge. Note that the Abelian limit is obtained by dropping all the multipole terms except the scalar component. The classical limit is obtained in the other limit, the so called large  $N_c$  limit, by admitting multipole contributions of arbitrary high orders. Finally, it may be noted that the two Casimirs  $Q^a Q^a$  and  $d^{abc} Q^a Q^b Q^c$  restrict the dynamics under the gauge independent interactions to a smaller subspace. As we shall see latter, the above resolution of the distribution functions brings out

the richness in the response functions.

### III. THE RESPONSE FUNCTIONS

#### A. The quark-antiquark sector

Strictly speaking, one should write a two particle distribution function for the  $q\bar{q}$  system to study the color charge excitations. This procedure is laborious [39] and we present a simplified version below. Consider color deviation of the  $q\bar{q}$  system from its equilibrium configuration  $f_0$ , which is taken to be a uniform distribution in color space. Accordingly, we write the distribution function in the form

$$f = f_0(p) + \hat{Q}^a f^a(x_i, p_i), \quad (2)$$

which emphasizes that  $f_0$  is independent of position and color variables. The color fluctuations which are obtained by perturbing around  $f_0$  are denoted by  $\hat{Q}^a f^a$ . The color density functions  $f^a$  are functions of both position and momentum. Finally, we also note that at equilibrium, the chromoelectric field vanishes identically.

The transport equation reads

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial p_i} + \dot{Q}^a \frac{\partial f}{\partial Q^a} = 0, \quad (3)$$

which on the application of the Wong equation [40]

$$\frac{dQ^a}{d\tau} = f^{abc} Q^b u_\mu A^{\mu c}, \quad (4)$$

acquires the form

$$\begin{aligned} \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + Q^a E_i^a(\vec{r}) \frac{\partial f}{\partial p_i} \\ - f^{lmn} \{A_0^l(\vec{r}) - v_i A_i^l(\vec{r})\} Q^m \frac{\partial f}{\partial Q^n} = 0, \end{aligned} \quad (5)$$

in obtaining which we have assumed that the plasma is spatially isotropic and that the chromomagnetic field is absent. Unlike the electrodynamic plasma, the above equation needs a specification of the gauge [41] [48]. We study Eq.5 in the temporal gauge,  $A_0^a = 0$ . In this gauge the chromoelectric field has a simple dependence on the gauge potential:

$$E_i^a(\vec{r}, t) = -\frac{\partial A_i^a(t, \vec{r})}{\partial t}.$$

The transport equation now gets a simpler form

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + Q^a E_i^a(\vec{r}) \frac{\partial f}{\partial p_i} + f^{lmn} v_i A_i^l(\vec{r}) Q^m \frac{\partial f}{\partial Q^n} = 0 \quad (6)$$

The evaluation of the chromoelectric response requires the determination of the equations for the moments of the distribution function, involving integrations in the

group space by employing the appropriate Haar measure in the group space. The details of the evaluation of the color moments are provided in the Appendix.

At this juncture, it is convenient to switch over to the vector notation for color variables:

$$f^a Q^a \equiv \vec{f} \cdot \vec{Q}; \quad f^{lmn} A^l B^m \equiv (\vec{A} \times \vec{B})^n.$$

The latter notation collapses to the usual notation when the gauge group is  $SU(2)$ . We continue to denote the position and momenta by their components. Thus, we write

$$\frac{\partial \vec{f}}{\partial t} + v_i \frac{\partial \vec{f}}{\partial x_i} + Q \vec{E}_i \frac{\partial f_0}{\partial p_i} - v_i \vec{A}_i \times \vec{f} = 0. \quad (7)$$

Note that in obtaining the above equation we have further assumed that  $\vec{f}$  is a small perturbation on  $f_0$ . In particular,  $\frac{\partial f_0}{\partial p_i}$  dominates over the gradient of the color vector fluctuation  $|\frac{\partial \vec{f}}{\partial p_i}|$  which we drop.

The color response function is to be derived from Eq.7 which, as we see, simply replaces the ordinary derivative by the covariant derivative, in the temporal gauge. It turns out that the extraction of the response function is not as straight forward as in the case of ordinary plasma. We follow the standard treatment and write Eq.7 in the Fourier space. We denote by  $\vec{\phi}(\omega, k_i, p_i)$ , the Fourier transform of  $\vec{f}(t, x_i, p_i)$ . Similarly,  $\vec{\mathcal{E}}_i$  and  $\vec{\mathcal{A}}_i$  represent the Fourier transforms of the gauge field and the gauge potential respectively. We obtain

$$\vec{\phi}(\omega, k_i, p_i) = -Q \vec{\mathcal{E}}_i \frac{1}{\mathcal{D}} \frac{\partial f_0}{\partial p_i} + (\vec{\mathcal{A}}_i \star \vec{\phi})(\omega, k_i, p_i) \frac{1}{\mathcal{D}} v_i. \quad (8)$$

In the above expression,  $\mathcal{D} = i\omega - ik_i v_i$ . We have employed the notation  $\star$  to denote the convolution of the Fourier transforms, combined with the cross product in the color variables:

$$(\vec{\mathcal{A}} \star \vec{\mathcal{B}})^c \equiv f^{abc} \int d\omega' d^3 k' A^a(\omega - \omega', k_i - k'_i) B^b(\omega', k'_i).$$

$Q$  denotes the magnitude of the color charge.

The expression for the charge density  $\vec{\rho}$  is easily obtained to be

$$\vec{\rho}(\omega, k_i) = -Q^2 \vec{\mathcal{E}}_i \int d^3 p \frac{1}{\mathcal{D}} \frac{\partial f_0}{\partial p_i} + Q (\vec{\mathcal{A}}_i \star \vec{\Phi}_i)(\omega, k_i), \quad (9)$$

where we have defined

$$\vec{\Phi}_i(\omega, k_i) = \int d^3 p v_i \frac{1}{\mathcal{D}} \vec{\phi}(\omega, k_i, p_i). \quad (10)$$

It is clear from the above equation that the expression for  $\vec{\rho}$  is not closed, even after we employ the Gauss Law expression, all because of the second term which cannot be written in terms of the charge density or the field. A rigorous evaluation would lead to an infinite hierarchy



of equations with moments of the distribution functions with respect to velocities. Since we are interested in the bulk properties of the medium, we evaluate Eq.9 in the long wavelength limit, by assuming that  $k_i/\omega \ll 1$ . In this approximation, Eq. 10 simply becomes

$$\vec{\Phi}_i(\omega, k_i) = \frac{1}{\omega} \vec{J}_i,$$

in terms of the color current density  $\vec{J}_i$ . It follows from Eq.8 that the equation for  $\vec{J}_i$  is given by

$$\vec{J}_i(\omega, k_i) = -Q^2 \vec{\mathcal{E}}_i \int d^3p \frac{1}{\mathcal{D}} v_i \frac{\partial f_0}{\partial p_i} + Q(\vec{\mathcal{A}}_i \star \vec{\Phi}_{ij})(\omega, k_i), \quad (11)$$

where

$$\vec{\Phi}_{ij}(\omega, k_i) = \int d^3p v_i v_j \frac{1}{\mathcal{D}} \vec{\phi}(\omega, k_i, p_i),$$

which again involves an expression which is one order higher in the moment with respect to velocity. We truncate the hierarchy, by dropping the contribution from the second term, and write

$$\vec{J}_i = -Q^2 \vec{\mathcal{E}}_i \int d^3p v_i \frac{1}{\mathcal{D}} \frac{\partial f_0}{\partial p_i}. \quad (12)$$

We may thus rewrite Eq.9 in the form

$$\vec{\rho}(\omega, k_i, p_i) = -iQ^2 \vec{\mathcal{E}}_i I_i - Q^2 \frac{1}{\omega} \vec{\mathcal{A}}_i \star (\vec{\mathcal{E}}_j I_{ij}). \quad (13)$$

The integrals  $I_i, I_{ij}$  are entirely functions of the equilibrium distribution functions, and given by

$$\begin{aligned} I_i &= \sqrt{-1} \int d^3p \frac{1}{\mathcal{D}} \frac{\partial f_0}{\partial p_i}, \\ I_{ij} &= \sqrt{-1} \int d^3p v_i \frac{1}{\mathcal{D}} \frac{\partial f_0}{\partial p_j}. \end{aligned} \quad (14)$$

The determination of permittivity is almost complete. Since we are considering an isotropic system, we observe that the integrals above have the resolutions

$$\begin{aligned} I_i &= k_i I_0(\omega, k) \\ I_{ij} &= \delta_{ij} I_1(\omega, k) + k_i k_j I_2(\omega, k). \end{aligned} \quad (15)$$

Clearly,  $I_2$  is analytic in  $k$  and, thus does not contribute at low wavelengths. The final expression is thus given by

$$\vec{\rho}(\omega, k_i) + iQ^2 k_i \vec{\mathcal{E}}_i(\omega, k_i) I_0(\omega, k) - \frac{Q^2}{\omega} \vec{\mathcal{A}}_i \star (\vec{\mathcal{E}}_i I_1) = 0. \quad (16)$$

The response functions can be inferred by comparing Eq.13 with the Gauss' law in vacuum in the Fourier space:

$$\vec{\rho}(\omega, k_i) - i k_i \vec{\mathcal{E}}_i(\omega, k_i) + \vec{\mathcal{A}}_i \star \vec{\mathcal{E}}_i = 0. \quad (17)$$

Combining Eqs.13 and 17, we are led to define the following permittivities

$$\epsilon_{ij}^{ab}(\omega, k) = \{1 + Q^2 I_0(\omega, k)\} \delta_{ij} \delta^{ab} \equiv \epsilon_A \delta_{ij} \delta^{ab}. \quad (18)$$

$$\epsilon_{ij}^{abc}(\omega, \omega') = \{1 + \frac{Q^2 I_1(\omega', k')|_{k'=0}}{\omega}\} f^{abc} \delta_{ij}$$

$$\equiv \epsilon_N f^{abc} \delta_{ij}. \quad (19)$$

Note the emergence of the new permittivity  $\epsilon_N$  which has no Abelian counterpart. It is an invariant tensor of rank three in the color space (being proportional to  $f^{abc}$ ). The new permittivity emerges, we stress, in addition to the Abelian permittivity  $\epsilon_A$ , which leads to the standard screening. As much as the Abelian permittivity signifies a renormalization of the charge, the non-Abelian permittivity exhibits the renormalization of the non-Abelian coupling coefficients  $f^{abc}$ . Gauge invariance of the theory constrains the two renormalizations, as shown in the above equations, via the dependence on  $f_{eq}$ . Explicitly, the two integrals  $I_0$  and  $I_1$  which define the two permittivities may be generated from a single function:

$$I_0 = \frac{1}{k^2} \frac{\partial}{\partial \omega} \int \ln(\mathcal{D}) k_i \partial_{p_i} f_{eq} d^3p \quad (20)$$

$$I_1 = -\frac{1}{k^2} \frac{\partial}{\partial k_i} \int \ln(\mathcal{D}) \partial_{p_i} f_{eq} d^3p. \quad (21)$$

Note that all the “couplings”  $f^{abc}$  undergo the same renormalization, as required by gauge invariance.

The resolution of the permittivity into the Abelian and the non-Abelian sectors is itself dependent on the gauge employed. The above identification is done in the temporal gauge. However, the results are themselves gauge invariant. We refrain from writing the response functions and the counter parts of Eq.19 in an arbitrary gauge as it is rather cumbersome, and of no use to us here.

## B. Form of the permittivities

It is instructive to study the form of the response functions in the simple case when  $f_{eq}$  has the standard Fermi-Dirac form. The expression for the Abelian permittivity reads

$$\epsilon_A = 1 + \frac{2\pi^3 Q^2 T^2 N_f}{3k^2} \left\{ -\frac{\omega}{k} \ln \left| \frac{\omega + k}{\omega - k} \right| + 2 \right\} \quad (22)$$

and is not different from that of the electrodynamic plasma, except for multiplicative color and flavor factors. The non-Abelian permittivity is however, novel, and has the form

$$\epsilon_N = \left\{ 1 - \frac{4\pi^3 Q^2 T^2 N_f}{9} \frac{1}{\omega \omega'} \right\}. \quad (23)$$

Although the functions are real, their imaginary components can be extracted by employing the standard Landau  $i\epsilon$  prescription. The imaginary components cause the Landau damping which we will discuss in detail in a separate section.

### C. Physical significance of $\epsilon_N$

The significance of  $\epsilon_N$  is best seen in the induced non-Abelian component of the charge density, the so called charge density carried by the field. In the Gauss' equation for non-Abelian fields the term  $\rho_a^f \equiv f^{abc} A_i^b E_i^c$  (where superscript 'f' stands for field component) has a natural interpretation of the charge density contributed by the field. This charge density is not gauge covariant. However, the total charge is gauge covariant, provided reasonable boundary conditions are imposed.  $\epsilon_N$  may now be looked upon as modifying  $\rho_a^f$  by inducing additional charges. To study this we evaluate the modification to  $\rho_a^f$  and obtain

$$\rho_a^f = -f^{abc} A_i^b(t, \vec{r}) E_i^c(t, \vec{r}) + \frac{\pi Q^2 T^2 N_f}{9} f^{abc} \times \int_{-\infty}^t dt' A_i^b(t', \vec{r}) \int_{-\infty}^{t'} dt'' E_i^c(t'', \vec{r}), \quad (24)$$

where the second term shows the induced charge density of the field. The above equation displays the inherently nonlinear, non-Markovian nature of the non-Abelian permittivity. In the collisionless limit that we are interested, the response is maximally non-Markovian.

It is pertinent at this stage to mention that the non-Abelian response plays an important role involving three gluon processes, especially gluonic bremsstrahlung of gluons, and the analog of Čerenkov radiation of gluons. Their phenomenological significance in heavy ion collisions remains to be investigated.

### D. The gluonic sector

We now consider the bosonic content of plasma. It differs from the matter sector in two respects. Its equilibrium distribution function is given by the Bose-Einstein form, and it has a richer structure in the color space. As a warm up, we first consider  $SU(2)$ , which is simpler.

#### 1. $SU(2)$ gluons

The color coherent states are no different from the more familiar spin coherent states (corresponding to spin -1) in the adjoint representation. The density operator in the coherent basis has the expansion

$$f(\vec{r}, \vec{p}, Q^a) = f_0 + \hat{Q}^a f^a + \hat{Q}^{ab} f^{ab}, \quad (25)$$

where the equilibrium distribution  $f_0$  corresponds to the singlet,  $f^a$  to the triplet  $D^1$ , and  $f^{ab}$  to the 5-dimensional irreducible representation  $D^2$  of  $SU(2)$ . Recall that  $f^{ab}$  is a completely symmetric and traceless matrix, as is also  $\hat{Q}^{ab}$  which is given by,

$$Q^2 \hat{Q}^{ab} = Q^a Q^b - \frac{\delta^{ab}}{3} Q^2.$$

There are, in all, nine independent functions of position and momentum. Of them, the tensor components  $f^{ab}$  are specific to the gluonic sector and are absent in the quark sector. Thus the perturbation around the equilibrium configuration has a richer structure for the gluons than for the quarks in the color space. We treat  $f^a$  and  $f^{ab}$  to be fluctuations around  $f_0$ .

We may now repeat the evaluation of the moments in the color space as in the quark sector. Since the tensors are irreducible, the quark sector results hold in the gluonic sector as well for the charge density leading essentially to the same expression for the permittivities

$$\epsilon_A = 1 + \frac{4\pi^3 Q^2 T^2}{3k^2} \left\{ -\frac{\omega}{k} \ln \left| \frac{\omega + k}{\omega - k} \right| + 2 \right\}, \quad (26)$$

$$\epsilon_N = 1 - \frac{8\pi^3 Q^2 T^2}{9} \frac{1}{\omega \omega'}. \quad (27)$$

These differ from the expressions for quarks because of the choice of  $f_0$ . However, unlike in the case of the quarks, the above permittivities *do not* capture completely the response of the medium. There are additional contributions coming from the tensor fluctuations  $f^{ab}$ . To study them, we evaluate the moment of the transport equation *wrt*  $Q^{ab}$ . In the spirit of the earlier calculations, we keep only the contribution from  $f_0$  in the terms involving the gradient *wrt* the momentum. In this case, it is preferable to evaluate the moment in the phase space, *i.e.*, without integration over momentum. Let us denote by  $\phi^{ab}(\omega, k_i, p_i)$  the Fourier transform of  $f^{ab}$ . We then get

$$\phi^{ab}(\omega, k_i, p_i) - \frac{i}{\omega} L^{abcde} v_i (\mathcal{A}_i^c \otimes \phi^{de})(\omega, k_i, p_i) = 0, \quad (28)$$

where the symbol  $\otimes$  denote only the convolution of the Fourier transforms, without any operation in the color space. The fifth rank invariant tensor is given by

$$L^{abcde} = i(\epsilon^{acd} \delta^{be} + \epsilon^{bcd} \delta^{ae}). \quad (29)$$

The above expression is an eigenvalue equation in the tensor fluctuations  $f^{ab}$ , in the extended phase space. The spectrum is, significantly, independent of the equation of state, *i.e.*, of  $f_0$  – within the approximate framework employed here. It is determined entirely by the structure constants and, of course, the perturbing gauge fields. It should be of interest to work out the experimental consequences, in terms of the propagation of gluons in the medium.

To gain further insight, we integrate Eq.28 over the momentum variables. We obtain

$$\rho^{ab}(\omega, k_i) - \frac{i}{\omega} L^{abcde} (\mathcal{A}_i^c \otimes J_i^{de})(\omega, k_i) = 0, \quad (30)$$

which establishes a relation between the tensor charge density

$$\int Q^2 \phi^{ab} d^3 \vec{p} = \rho^{ab},$$

with the tensor current density

$$\int Q^2 v_i \phi^{ab} d^3 \vec{p} = J_i^{ab}.$$

Observe that Eq.30 bears some resemblance to the continuity equation, except that it does not have the standard convective term. The absence may be attributed to the long wavelength approximation. The corresponding tensor charge would only be approximately conserved.

## 2. The $SU(3)$ gluons

The generalization to the  $SU(3)$  case is not difficult. The form of the distribution function is given by

$$f(x_i, p_i, \vec{Q}) = f_0 + \hat{Q} \cdot \vec{f} + \hat{Q}^{ab} f^{ab}, \quad (31)$$

which is formally identical to Eq.25. The transformation properties are however different:  $f_0$  is the singlet,  $f^a$  the octet, and the tensor  $f^{ab}$  is the 27-plet of  $SU(3)$ . The structure of the tensor basis  $\hat{Q}^{ab}$  is more complicated than its counterpart in  $SU(2)$  [42]:

$$Q^2 \hat{Q}^{ab} = Q^a Q^b - \frac{3}{5} d^{pqr} d^{rab} Q^p Q^q - Q^2 \frac{\delta^{ab}}{8}, \quad (32)$$

where  $d^{abc}$  are the symmetric structure constants of  $SU(3)$ . Note that  $\hat{Q}^{ab}$  is symmetric and traceless. In all, there are  $1+8+27$  parameters that characterize the color distribution, as it should be for a  $8 \times 8$  density operator. We shall not write the expressions for the Abelian and the non-Abelian permittivities since they are identical to Eq.26 and Eq.27.

At this juncture we note that there is an interesting relation between the permittivities of quarks and gluons obeying the ideal equation of state. Consider the susceptibilities  $\mathcal{A}(q, g) = \epsilon_A - 1$  and  $\mathcal{N}(q, g) = \epsilon_N - 1$  for the quarks(q) and the gluons(g). It can be easily verified that the following relation holds

$$\begin{aligned} \mathcal{A}(q) &= \frac{N_f}{2} \mathcal{A}(g), \\ \mathcal{N}(q) &= \frac{N_f}{2} \mathcal{N}(g). \end{aligned} \quad (33)$$

It can be shown that it is true for any  $SU(N)$  gauge group.

We turn our attention to the tensor fluctuations  $f^{ab}$ . They satisfy the eigenvalue equations

$$R^{abcd} \phi^{cd}(\omega, k_i, p_i) - 2i L^{abcde} v_i \frac{1}{\mathcal{D}} \mathcal{A}_i^c \otimes \phi^{de} = 0, \quad (34)$$

where the rank four tensor, specific to  $SU(3)$  has the form

$$R^{abcd} = (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc} - \frac{6}{5} d^{abe} d^{ecd}),$$

and the rank five tensor has the form

$$L^{abcde} = (f^{acd} \delta^{be} + f^{bcd} \delta^{ae} - \frac{6}{5} f^{mce} d^{abr} d^{rdm}).$$

A further integration over momentum yields, in the large wavelength limit,

$$R^{abcd} \rho^{cd}(\omega, k_i) - 2 \frac{i}{\omega} L^{abcde} \frac{1}{\mathcal{D}} \mathcal{A}_i^c \otimes J_i^{de} = 0 \quad (35)$$

which is similar to Eq.30. It has a more complicated structure with the occurrence of the symmetric as well as the antisymmetric structure constants of the group. We study the above equations in a little bit more detail in the next section.

We wish to point out that one needs rather special fields to excite the tensor fluctuations. To demonstrate this aspect, let us consider Eq.35 when the gauge fields represent a plane wave, i.e.,

$$A_i^a(\omega, k_i) = -i \frac{\vec{E}_{0i}^a}{\omega} \delta(\omega - \omega_0) \delta^3(k_i - k_{i0}).$$

For this choice, Eq.35 has only the trivial solution, viz,  $f^{ab} \equiv 0$ . One may expect that the tensor fluctuations may be excited provided the field has a quadrupole component for  $SU(2)$ , and its generalization thereof for  $SU(3)$ . The implication of this result to the signatures of QGP needs to be explored.

## 3. Closure in the color space

We have seen that the color degree of freedom does not possess an infinite hierarchy of moments since its associated phase space is compact. For  $SU(2)$ , the hierarchy of the moment equations terminates at the dipole level for the matter sector, and at the quadrupole level for the gluonic sector, as may be seen below:

$$\phi^b + iQ \frac{1}{\mathcal{D}} \mathcal{E}_i^b \frac{\partial f_0}{\partial p_i} - i v_i \frac{1}{\mathcal{D}} (\vec{\mathcal{A}}_i \star \vec{\phi})^b + \frac{3iV_2}{V_1} \frac{1}{\mathcal{D}} \mathcal{E}_i^a \otimes \frac{\partial \phi^{ab}}{\partial p_i} = 0, \quad (36)$$

where  $V_1, V_2$  are defined in the Appendix. Similarly, the tensor fluctuations satisfy

$$\phi^{ab} + iQ \frac{1}{\mathcal{D}} \{ \mathcal{E}_i^a \otimes \frac{\partial \phi^b}{\partial p_i} + \mathcal{E}_i^b \otimes \frac{\partial \phi^a}{\partial p_i} \} - iL^{abcde} \frac{v_i}{\mathcal{D}} \mathcal{A}_i^c \otimes \phi^{de} = 0. \quad (37)$$

The above equations are written without making the long wavelength approximation. This closure as exhibited in Eqs.36, 37 is formal since the gauge fields have to be determined self consistently with the matter distribution,

as governed by the Yang-Mills equations.

The corresponding equations for  $SU(3)$  are slightly more complicated:

$$\phi^b + iQ \frac{1}{\mathcal{D}} \mathcal{E}_i^b \frac{\partial f_0}{\partial p_i} - iv_i \frac{1}{\mathcal{D}} (\vec{\mathcal{A}}_i \star \vec{\phi})^b + \frac{16iV_2}{V_1} \frac{1}{\mathcal{D}} \mathcal{E}_i^a \otimes T^{abcd} \frac{\partial \phi^{cd}}{\partial p_i} = 0 \quad (38)$$

where the tensor

$$T^{abcd} = \delta^{ac} \delta^{bd} - \frac{3}{5} d^{abe} d^{ecd}.$$

The tensor excitations satisfy the equation

$$R^{abcd} (\phi^{cd} + iQ \frac{1}{\mathcal{D}} \{ \mathcal{E}_i^c \otimes \frac{\partial \phi^d}{\partial p_i} + \mathcal{E}_i^d \otimes \frac{\partial \phi^c}{\partial p_i} \}) - iL^{abcde} \frac{v_i}{\mathcal{D}} \mathcal{A}_i^c \otimes \phi^{de} = 0 \quad (39)$$

where the two constants  $V_1$  and  $V_2$  are defined in appendix A. We see again that the above two equations constitute a closed set, in a formal sense. Although their full solution in a self consistent manner is an intricate and an involved exercise, the clear signal from the above equations is that generically, all the fluctuations exist and none of the responses can be ignored. It also shows that if required, it is possible to improve upon our long wavelength approximation in a systematic manner.

## IV. RESULTS AND DISCUSSION

### A. The Abelian response

Our analysis has made no assumption on the equilibrium distribution, save its isotropy in the momentum space, and independence of the position coordinates. We now consider, purely for purposes of illustration, an ideal gas of quarks and gluons. Of course, the Abelian component of the chromoelectric response is no different from its electrodynamic counterpart, except for color-spin factors. We merely record two of its properties.

Considering quarks first, we get

$$\lim_{\omega \rightarrow 0} \epsilon^A(\omega, k) = (1 + \frac{m_D^2}{k^2}) \quad (40)$$

$$\lim_{k \rightarrow 0} \epsilon^A(\omega, k) = (1 - \frac{m_D^2}{3\omega^2}) \quad (41)$$

written in terms of the standard Debye mass

$$m_D^2 = 4\pi^3 Q^2 T^2 N_f / 3.$$

The corresponding expressions for the gluonic plasma is obtained by simply replacing  $N_F$  by 2 in the expression for  $m_D^2$ . It should be borne in mind that since we either have a gluonic plasma or a quark gluon plasma, the expression for  $m_D^2$  for QGP is obtained by averaging over the contributions from the quarks and gluons, weighted by the respective relative densities.

It is commonly accepted that the inverse Debye mass yields the screening length for the heavy quark potentials. This is so for electrodynamic plasmas, governed by the Coulomb potential, which goes over to Yukawa. The situation is not that simple in the case of quarkonia. Since  $J/\Psi$  suppression is one of the strong signatures for QGP, it is worth analyzing the import of the Debye mass to realistic heavy quark potentials. We take it up in the next subsection.

### B. The heavy quark potential

Consider the Cornell potential given by [43]

$$\phi(r) = -\frac{\alpha}{r} + \Lambda r,$$

where  $\alpha$  and  $\Lambda$  are phenomenological constants. This potential has been studied in depth by Brambilla *et al*



[44]. The potential, has a Coulombic behavior at small distances and a linearly confining form at large distances. Note that the parameter  $\Lambda$  has mass dimension 2 ( $\alpha$  has mass dimension 0). The medium alters the above potential as given by the modified expression for the potential (in the Fourier space)  $\phi(k) \rightarrow \phi_s(k) = \phi(k)/\epsilon(k)$ .  $\phi_s(k)$  is obtained to be

$$\phi_s(k) = -\sqrt{\frac{2}{\pi}} \frac{\alpha}{k^2 + m_D^2} - \frac{4}{\sqrt{2\pi}} \frac{\Lambda}{k^2(k^2 + m_D^2)} \quad (42)$$

The inverse Fourier transform of the above expression yields the modified potential, as a function of distance, as shown below:

$$\phi_s(r) = \left(\frac{2\Lambda}{m_D^2} - \alpha\right) \frac{\exp(-m_D r)}{r} - \frac{2\Lambda}{m_D^2 r}. \quad (43)$$

The above expression merits a closer scrutiny. Interestingly, the Coulomb term goes over to the short range Yukawa form while the linear confining term goes over to a sum of Yukawa and the long range Coulomb potential. The Coulombic tail, which is relevant at large distances, supports – as is universally known – an infinite number of bound states. The effective Coulombic charge is given by  $\frac{2\Lambda}{m_D^2}$ ; this suggests that the role played by the Debye mass cannot be naively reduced to the notion of a screening length. Consequently, the relevant parameter to study the deconfinement is, *not the inverse Debye mass*, but the dissociation energy as a function of the parameter  $\frac{2\Lambda}{m_D^2}$ . Note that it is also the relevant parameter at high temperature.

Thus in the large distance/high temperature limit, it is not difficult to estimate the dissociation energy  $\mathcal{E}_D$ . We obtain

$$\mathcal{E}_D \sim m_q \Lambda^2 / m_D^4 \equiv \frac{3^2}{(4\pi^3)^2} \frac{m_q \Lambda^2}{Q^4 T^4 N_F^2}. \quad (44)$$

It is further possible to estimate the temperature  $T_D$  at which the dissociation takes place. Employing the equipartition theorem, we obtain

$$T_D^5 = \frac{6}{(4\pi^3)^2} \frac{m_q \Lambda^2}{N_F Q^2}. \quad (45)$$

We note that for a purely gluonic plasma, the factor  $N_F$  gets replaced by 2.

In short, a rather straightforward and a simple analysis has allowed us to extract the precise physical significance of the Debye mass for the dissociation, or equivalently, the suppression of the charmonium states in QGP. It also demonstrates that care should be exercised in interpreting the inverse mass as a screening length. The physics is dictated by a combination of two scales,  $\Lambda$  and  $m_D^2$ , as obtained in Eq.43.

### C. The non-Abelian response

We turn our attention to the non-Abelian permittivity. In the long wavelength limit, it has a simple expression

given by

$$\epsilon_N = 1 - \frac{m_D^2}{3\omega\omega'}.$$

It is interesting to note that the above expression is almost identical to the expression for the Abelian permittivity at  $k = 0$ . In fact, the quantity ( $\omega^2$  in  $\epsilon_A$  gets simply replaced by  $\omega\omega'$  in  $\epsilon_N$ ). The non-Abelian response vanishes at large  $\omega, \omega'$ , and the response gets stronger with temperature. Of course, it vanishes when  $T = 0$ , but it cannot be taken seriously since the interaction effects would dominate at small temperatures. As mentioned, we may expect the non-Abelian response to play significant role in processes involving three gluon vertices.

### V. LANDAU DAMPING IN QGP

Landau damping in QGP has been studied earlier by Heinz and Siemens [45] and Murtaza, Khattak and Shah [46]. We obtain here an explicit expression for Landau damping in QGP, arising especially from the non-Abelian response. A naive extension of the standard formula will not do since we have additional permittivities. Further the Abelian and non-Abelian components of the permittivities are not independent because of gauge invariance. We address the problem *ab initio*.

The interaction Hamiltonian is given by

$$\mathcal{H} = J_\mu^a A^{a\mu}. \quad (46)$$

The energy dissipation per unit time per unit volume has the standard expression

$$\mathcal{Q} = -\vec{J}_i \cdot \vec{E}_i.$$

Recall that we are in the temporal gauge. Employing Eq.12, which we reproduce below,

$$\vec{J}_i = -Q^2 \vec{\mathcal{E}}_j \int d^3 p v_i \frac{1}{D} \frac{\partial f_0}{\partial p_j},$$

it is straight forward to obtain the contribution to the energy density due to the non-Abelian response. Finally, adding the standard Abelian contribution, the total energy density per unit time of an electric field is obtained to be

$$\mathcal{U} = \omega \{1 + Q^2 I_0(\omega, k)\} \vec{E}_i \cdot \vec{E}_i + Q^2 I_1(\omega, k) |_{k=0} \vec{E}_i \cdot \vec{E}_i. \quad (47)$$

In Eq.47 the contribution from imaginary term gives the rate of the energy dissipation rate per unit volume,  $\mathcal{E}_{dis}$ . The contributions from  $I_0$  and  $I_1$  are respectively from the Abelian and the non-Abelian permittivities. It is noteworthy that in the collisionless limit that we are interested in, only the Abelian component contributes to the damping, for, Eq.47 leads to the explicit form

$$\mathcal{U}_{dis} = \pi m_D^2 \left\{ \frac{\omega^2}{2k^3} \theta(k - \omega) + \frac{\delta(\omega)}{3} \right\} \vec{E}_i \cdot \vec{E}_i. \quad (48)$$

The non-Abelian contribution is saturated at  $\omega = 0$  and is hence not an observable.

## VI. CONCLUSION

In conclusion, we have developed a systematic formulation for determining the chromoelectric response of QGP within the classical framework. We have shown the emergence of a non-Abelian component of the response, characterized by new permittivity  $\epsilon_N$ .  $\epsilon_N$  is, unlike its Abelian component  $\epsilon_A$ , nonlocal and non-Markovian. Furthermore we have also derived explicitly the response functions of gluons which are shown to be richer than that of the quarks. The precise role of the Debye mass in the so called screening of the heavy quark potential is extracted. Nevertheless, we point out that the applications discussed in this paper are only indicative in nature since we have ignored the strongly interacting nature of QGP; our expressions for the equilibrium distribution functions are highly approximate. An application of the analysis made in this paper, with realistic equations of state as applied to QGP will be taken up separately.

## VII. APPENDIX

### A. Color space integrals

The Haar measure for the SU(3) group is given by

$$dQ = d^8Q \delta(Q^a Q_a - q^2) \delta(d^{abc} Q^a Q^b Q^c - q^3)$$

[47], where  $q^2 = N$  and  $q^3 = 0$  in the adjoint representation of SU(N) group [30]. Some useful color integrations in SU(3) group space are given by

$$\int Q^a dQ = 0; \quad \int Q^{ab} dQ = 0$$

$$\int Q^c Q^{ab} dQ = 0; \quad \int Q^a Q^b dQ = \frac{\delta^{ab}}{8} Q^2 V_1;$$

$$\int Q^a Q^b Q^c dQ = 0$$

$$\int Q^a Q^b Q^c Q^d dQ = V_2 Q^4 (\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}).$$

where  $V_1$  is the volume of color space and  $V_2$  is a constant.  $Q^{ab}$  has been defined as (see Eq.32)

$$Q^{ab} = Q^a Q^b - \frac{3}{5} d^{pqr} d^{rab} Q^p Q^q - \frac{\delta^{ab}}{8} Q^2.$$

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- [1] STAR collaboration, J. Adams *et al.* Nucl. Phys. A **757** 102(2005).
  - [2] Itzhak Tserruya, Nucl. Phys. A **774** 415(2006), Nucl.Phys.A **774** 433(2006).
  - [3] Patricia Fachini, AIP Conf. Proc. **857** 62(2006).
  - [4] Steffan A. Bass, Pramana **60** 593(2003).
  - [5] Helmut Satz, Rep. Prog. Phys. **63** 1511(2000).
  - [6] T. Csörgö, S. Hegyl, T. Novak and W. A. Zajc, Acta Phys. Polon. B **36** 329(2005).
  - [7] P. K. Kovtun, D.T. Son and A.O. Starinets, Phys. Rev. Lett. **94** 111601(2005).
  - [8] M. J. Tannenbaum, Rept. Prog. Phys. **69** 2005(2006).
  - [9] Frithjof Karsch, Nucl. Phys. A **698** 199(2002).
  - [10] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Proceedings of PoS LAT2005 163(2006).
  - [11] K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, Phys. Rev. D **67** 105008(2003).
  - [12] Ulrich Heinz, Proceedings of \*Swansea 2005, Extreme QCD\* 3-12.
  - [13] R. D. Pisarski, Phys. Rev. Lett. **63** 1129(1989).
  - [14] E. Braaten and R. D. Pisarski, Nucl. Phys. B **337** 569(1990).
  - [15] E. Braaten and R. D. Pisarski, Nucl. Phys. B **339** 310(1990).
  - [16] J. P. Blaizot and E. Iancu, Nucl. Phys. B **417** 608(1994).
  - [17] J. P. Blaizot and E. Iancu, Phys. Rev. Lett. **72** 3317(1994).
  - [18] J. P. Blaizot and E. Iancu, Nucl. Phys. B **421** 565(1994).
  - [19] K. Geiger, Phys. Rep. **258** 237(1995).
  - [20] A. Kovner, L. McLerran and H. Weigert, Phys.Rev. D **52** 3809(1995).
  - [21] Gouranga C Nayak and V. Ravishankar, Phys. Rev. C **58** 356 (1998); Phys. Rev. D **55** 6877 (1997).
  - [22] F. Gelis and R. Venugopalan, Lectures given at Cracow School of Theoretical Physics: 46th Course 2006, Zakopane, Poland, 27 May - 6 Jun 2006, hep-ph/0611157.
  - [23] D. Bodeker, Phys. Lett. B **559** 502(1999).
  - [24] T. Matsui and H. Satz, Phys. Lett. B **178** 416(1986).
  - [25] H. A. Weldon, Phys. Rev. D **26** 1394(1982).
  - [26] Munshi G. Mustafa, Markus H. Thoma and Purnendu Chakraborty, Phys. Rev. C **71** 017901(2005).
  - [27] Jinfeng Liao and E. V. Shuryak, Nucl. Phys. A **775** 224(2006).
  - [28] P ter Petreczky, hep-ph/9907247.
  - [29] Ji-sheng Chen, Jia-rong Li and Peng-fei Zhuang, Int. J. Mod. Phys. A **17** 1435(2002).
  - [30] Daniel F. Litim and Cristina Manuel, Phys. Rep. **364** 451(2002).
  - [31] Ambar Jain and V. Ravishankar, Phys. Rev. Lett. **91** 112301(2003).
  - [32] R. S. Bhalerao and V. Ravishankar, Phys. Lett. B **409** 38 (1997)
  - [33] S. M. Schmidt et.al., Phys. Rev. D **59** 094005 (1999)
  - [34] P. L. Bhatnagar, M. Gross and M. Crook, Phys. Rev. **94** 511 (1954)
  - [35] Ulrich Heinz, Phys. Rev. Lett. **51** 351(1983).
  - [36] H. T. Elze and Ulrich Heinz, Phys. Rep. **183** 81(1989).
  - [37] A. Perelomov, Generalized Coherent States and Their

- Applications, Springer Verlag (Berlin, Germany) 1986.
- [38] Mark Byrd, physics/9708015.
  - [39] Akhilesh Ranjan, thesis (2008) unpublished
  - [40] S. K. Wong, Nuovo Cimento A **65** 689(1970).
  - [41] Tai Tsun Wu and Chen Ning Yang Phys.Rev. **D13** 3233 (1976).
  - [42] A. J. Macfarlane, A. Sudbery and P. H. Weisz, Commun. math. Phys. **11** 77(1968).
  - [43] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **21** 203(1980).
  - [44] N. Brambilla *et al*, CERN Yellow Report, CERN-2005-005, Geneva: CERN, 2005.- 487 p.
  - [45] Ulrich Heinz and Philip J. Siemens, Phys. Lett. B **158** 11(1985).
  - [46] G. Murtaza, N. A. D. Khattak and H. A. Shah, Phys. Rev. E **68** (2003) 066404.
  - [47] Ulrich Heinz, Annals of Physics, **161** 48(1985).
  - [48] Unlike as in electrodynamics, the gauge invariant content of the field is not exhausted by just giving the field tensor. In fact, the same field tensor can be obtained by inequivalent gauge potentials [41]. Hence the gauge potential in Wong equation Eq.4 has nontrivial consequences.